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Properties of a consistent Lorentz-violating Abelian gauge theory

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Abstract

A Lorentz violating modification of massless *QED* is proposed, with higher order space derivatives for the photon field. The fermion dynamical mass generation is studied with the Schwinger-Dyson approach. Perturbative properties of the model are calculated at one-loop and discussed at higher-order loops, showing the consistency of the model. We explain that there is no contradiction with the definition of the speed of light c , although fermions see an effective light cone, with a maximum speed smaller than c .

1 Introduction

Recently, field theories with higher-order space derivatives have attracted attention, because of the improvement of graph convergence [1], at the price of violating Lorentz symmetry at high energies. Ghosts are not introduced by this procedure, since the time derivative order remains minimal, such that no new pole appears in the propagator of particles. Also, theories based on anisotropic scaling of space and time (Lifshitz type theories) allow new renormalizable interactions, and for example a renormalizable exponential potential, in 3+1 dimensions, has been studied in [2]. Finally, a renormalizable Lifshitz type theory of Gravity has been proposed, which could lead to

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Quantum Gravity [3].

A Lorentz-violating extension of massless Quantum Electrodynamics (QED) is discussed here, where higher order space derivatives are introduced for the photon field. This model has isotropic scaling in space time though, such that the higher order space derivatives involve a mass scale M , whose role will be discussed a bit further. Higher order space derivatives were already considered in a modified Dirac equation [4], where the resulting phenomenology concerning gamma ray bursts is also discussed. Higher order space derivatives could arise, for example, from quantum gravitational space time foam [5]. The mass M naively suppresses the effect of higher order derivatives in the IR, but as we will see, an IR signature of these higher orders remains: a fermion mass is generated dynamically. This mass, although proportional to M , is orders of magnitude smaller than M since it is suppressed by an exponentially small function of the fine structure constant. We will study here this mass generation, using the non-perturbative Schwinger-Dyson approach, and we will discuss the properties of the result. We note here that another alternative to the Higgs model has been proposed in the context of anisotropic theories [6], and dynamical mass generation has been studied in [7] for a Lifshitz-type four-fermion model, and in [8] for a Lifshitz type Yukawa model.

We mention here the first works on dynamical mass generation in QED [9], where a relation between the bare and dressed masses is derived, which involves a cut off Λ . In this context, it was shown that, in order to have a finite theory, the limit $\Lambda \rightarrow \infty$ implies that the bare mass should vanish, such that the dressed mass must be of dynamical origin. The Schwinger Dyson equation with a finite cut off was studied later [10], where a critical value for the fine structure constant was found in order to have dynamical mass generation in the limit $\Lambda \rightarrow \infty$. A summary of these results can be found in [11].

Once the fermion dynamical mass generation is demonstrated, we study the perturbative quantization of the model, taking into account the fermion mass. We show that the model leads to the same renormalization flows as QED , when one considers the running scale M . The only difference is the effective light cone seen by fermions, since space a time derivatives get different quantum corrections. We find that the maximum speed for fermions is smaller than the speed of light, by terms of one-loop order, showing that the model is consistent with causality. This is not in contradiction with the definition speed of light, since the latter is obtained in the massless limit, which cannot

be taken for fermions, due to dynamical mass generation. The speed of light is given by the dispersion relation for the gauge field, which is not altered by the Lorentz-violating model proposed here.

We note finally that more general higher order derivative extensions of *QED* have been presented in [12] and references therein. In these works, the authors consider Lorentz-violating vacuum expectation values for tensor fields, which allow the introduction of higher order derivatives for the photon field. They explain that the Lorentz-violating Lagrangians can be written as the Lagrangian for *QED* in a medium, and they study for example the corresponding birefringence effects of the vacuum. Our present study corresponds to a specific case of the latter models, where we develop quantum properties of a given Lorentz-violating extension of *QED*.

In Section 2 we define the model and discuss some of its classical features. The fermion dynamical mass is then derived in Section 3, and the properties of the quantum theory are developed in Section 4, where we first focus on one-loop, and then develop general arguments for higher-order loops. The Conclusion presents future extensions involving non-Abelian gauge fields, and an alternative to the Higgs mechanism. Finally, the Appendix contains the details of one-loop calculations.

2 Model

The Lorentz-violating Lagrangian considered here is

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu} \left(1 - \frac{\Delta}{M^2}\right) F_{\mu\nu} - \frac{\xi}{2}\partial_\mu A^\mu \left(1 - \frac{\Delta}{M^2}\right) \partial_\nu A^\nu + i\bar{\psi} \not{D}\psi, \quad (1)$$

where $D_\mu = \partial_\mu + ieA_\mu$, and $\Delta = -\partial_i\partial^i = \vec{\partial} \cdot \vec{\partial}$ (the metric used is (1,-1,-1,-1)), which recovers *QED* in a covariant gauge if $M \rightarrow \infty$. The Lorentz-violating terms have two roles: introduce a mass scale, necessary to generate a fermion mass, and lead to finite gap equation, as will be seen further. We stress here that M regularizes only loops with an internal photon line, and that another regularization is necessary to deal with fermion loops. Also, the Lorentz violating modifications proposed in the Lagrangian (1) do not alter the photon dispersion relation, which remains relativistic.

No higher order space derivatives are introduced for the fermions, for the following reason: in order to respect gauge invariance, such terms would

need to be of the form

$$\frac{1}{M^{n-1}} \bar{\psi} (i \vec{D} \cdot \vec{\gamma})^n \psi \quad n \geq 2, \quad (2)$$

such that new and non-renormalizable couplings would be introduced.

Equations of motion

The classical equation of motion for the free gauge field is

$$\left(1 - \frac{\Delta}{M^2}\right) (\Box \eta_{\mu\nu} - \partial_\mu \partial_\nu) A^\mu = 0, \quad (3)$$

and has two solutions. One is the usual plane wave, with the usual dispersion relation, and the other is solution of

$$\left(1 - \frac{\Delta}{M^2}\right) A_\mu = 0. \quad (4)$$

If the solution depends on time and one spatial direction x only, it reads

$$A_\mu = K_\mu(t) e^{-Mx}, \quad (5)$$

where $K_\mu(t)$ is an homogeneous vector. If we consider a spherically symmetric solution, depending on time and the radial spatial coordinate r only, we find

$$A_\mu = K_\mu(t) \frac{e^{-Mr}}{r}. \quad (6)$$

In both cases, the only regular solution occurs when $K_\mu = 0$. As a consequence, the only physical solution of the classical equation of motion for the free gauge field is the usual plane wave.

Lorentz transformation

Finally, one can ask what the effect of a Lorentz transformation is on higher order space derivatives, as in the model (1). For this, we consider two coordinate systems (t, \vec{r}) and (t', \vec{r}') , with the following transformation law

$$\begin{aligned} t &= \gamma(t' + \vec{v} \cdot \vec{r}') \\ \vec{r} &= \vec{r}_\perp' + \gamma(\vec{r}_\parallel' + \vec{v} t'), \end{aligned} \quad (7)$$

where the usual notations are used. In the Lorentz transformation (7), we have

$$-\partial_t^2 + \Delta = -\partial_{t'}^2 + \Delta' \quad \text{with} \quad \partial_{t'} = \gamma \partial_t + \gamma \vec{v} \cdot \vec{\nabla} , \quad (8)$$

such that

$$\Delta' = \Delta + (\gamma^2 - 1)\partial_t^2 + 2\gamma^2 \vec{v} \cdot \vec{\nabla} \partial_t + \gamma^2 (\vec{v} \cdot \vec{\nabla})^2 . \quad (9)$$

Using eq.(9), the equation of motion $(M^2 - \Delta')\phi(t', \vec{r}') = 0$ for a scalar field

$$\phi(t', \vec{r}') = \phi(t, \vec{r}) = \phi_0 \exp(i\omega t - i\vec{k} \cdot \vec{r}) , \quad (10)$$

leads then to the dispersion relation

$$\omega^2 + 2\omega \frac{\vec{v} \cdot \vec{k}}{v^2} + \frac{(\vec{v} \cdot \vec{k})^2}{v^2} + \frac{M^2 + k^2}{(v\gamma)^2} = 0 , \quad (11)$$

such that

$$\omega = -\frac{\vec{v} \cdot \vec{k}}{v^2} \pm \frac{i}{v\gamma} \sqrt{M^2 + k^2 - (\vec{v} \cdot \vec{k})^2 / v^2} . \quad (12)$$

As a consequence, the corresponding solution is not observable, since it decays in a time of the order of the Plank time (we do not take into account the non-physical solution increasing exponentially in time). Therefore the higher order time derivatives introduced by a Lorentz transformation do not affect the model presented here.

3 Gap equation

We review here the results of [13]. The Schwinger-Dyson equation for the fermion propagator is [11]:

$$G^{-1} - G_{bare}^{-1} = \int D_{\mu\nu}(e\gamma^\mu)G\Gamma^\nu , \quad (13)$$

where Γ^ν , G and $D_{\mu\nu}$ are respectively the dressed vertex, the dressed fermion propagator and the dressed photon propagator. This equation gives an exact self consistent relation between dressed n -point functions, and thus is not perturbative. As a consequence, no redefinition of bare parameters can be done in order to absorb would-be divergences, and for this reason one needs this equation to be regularized by M .

In the studies of dynamical mass generation in *QED* in the presence of an

external magnetic field B (“magnetic catalysis” [14]), the gap equation, in the Lowest Landau Level approximation, is finite because of the mass scale $\sqrt{|eB|}$ - where e is the electric charge - which plays the role of a gauge invariant cut off. Another example of dynamical mass generation, however of a different nature, is the Debye screening for the photon, at finite temperature, which was found to be enhanced by a strong magnetic field [15].

In order to solve the Schwinger-Dyson equation (13), we consider the so-called ladder (or rainbow) approximation, consisting in taking the bare vertex instead of Γ^ν . It is known that this approximation is not gauge invariant [11], but, as we will see, the gauge coupling dependence of the dynamical mass is not affected by the choice of the gauge parameter ξ . Then we will neglect corrections to the photon propagator, which is given by

$$D_{\mu\nu}^{bare}(\omega, \vec{p}) = \frac{i}{1 + p^2/M^2} \left(\frac{\eta_{\mu\nu}}{-\omega^2 + p^2} + \zeta \frac{p_\mu p_\nu}{(\omega^2 - p^2)^2} \right), \quad (14)$$

where $\zeta = 1/\xi - 1$, $p_0 = \omega$ and $p^2 = \vec{p} \cdot \vec{p}$. Also, we neglect the fermion wave function renormalization: only the fermion dynamical mass will be taken into account as a correction, such that the dressed fermion propagator will be taken as

$$G(\omega, \vec{p}) = i \frac{\omega \gamma^0 - \vec{p} \cdot \vec{\gamma} + m_{dyn}}{\omega^2 - p^2 - m_{dyn}^2}, \quad (15)$$

where m_{dyn} is the fermion dynamical mass. With these approximations, the Schwinger-Dyson equation (13) - involving a convergent integral - leads to

$$m_{dyn} = \frac{\alpha}{\pi^2} \int_{-\infty}^{\infty} d\omega \int_0^{\infty} \frac{p^2 dp}{1 + p^2/M^2} \frac{m_{dyn}(4 + \zeta)}{(\omega^2 + p^2)(\omega^2 + p^2 + m_{dyn}^2)}, \quad (16)$$

where $\alpha = e^2/4\pi$ is the fine structure constant. This equation has the obvious solution $m_{dyn} = 0$, and potentially a second solution, which must satisfy the following gap equation, obtained after integration over the frequency ω ,

$$\frac{\pi}{(4 + \zeta)\alpha} = \int_0^{\infty} \frac{x dx}{1 + \mu^2 x^2} \left(1 - \frac{x}{\sqrt{1 + x^2}} \right), \quad (17)$$

where $\mu = m_{dyn}/M$ is the dimensionless dynamical mass, expected to be very small $\mu \ll 1$, and $x = p/m_{dyn}$. Both terms in the last equation, if taken separately, lead to diverging integrals, with divergences which cancel each

other. An integration by parts for the second term leads then to

$$\begin{aligned}
\frac{2\pi}{(4+\zeta)\alpha} &= \frac{1}{\mu^2} \int_0^\infty dx \frac{\ln(1+\mu^2 x^2)}{(1+x^2)^{3/2}} \\
&= \frac{2}{\mu^2} \left(\ln\left(\frac{\mu}{2}\right) + \frac{\cosh^{-1}(1/\mu)}{\sqrt{1-\mu^2}} \right) \\
&= \ln\left(\frac{2}{\mu}\right) - \frac{1}{2} + \mathcal{O}(\mu^2 \ln \mu)
\end{aligned} \tag{18}$$

and the fermion dynamical mass is finally given by

$$m_{dyn} \simeq M \exp\left(-\frac{2\pi}{(4+\zeta)\alpha}\right). \tag{19}$$

Note that the expression (19) for m_{dyn} is not analytic in α , such that a perturbative expansion cannot lead to such a result, which justifies the use of a non-perturbative approach. A perturbative expansion would lead to the solution $m_{dyn} = 0$ only.

Among the two solutions $m_{dyn} = 0$ and $m_{dyn} \neq 0$, the physical system chooses the non-vanishing dynamical mass, in order to avoid IR instabilities, not favorable energetically, which would otherwise occur in the theory.

Gauge dependence

There is an obvious dependence on the gauge parameter ζ , which has a consequence on the value of m_{dyn} , but the important point is the non-analytic α -dependence of the dynamical mass, which is not affected by the choice of gauge: the resulting dynamical mass is of the form $M \exp(-c/\alpha)$, where c is a constant of order 1. This feature is known in the studies of dynamical mass generation in *QED* in the presence of an external magnetic field [14]. It has been argued though, using the pinch technique [16], that for the calculation of physical quantities, which must be gauge invariant, one should use the Feynman gauge $\zeta = 0$ to calculate the fermion self energy. The reason for this is the cancellation of terms arising from longitudinal contributions in the photon propagator. Nevertheless, if M is the Plank mass, the Feynman gauge for the dynamical mass (19) leads to a too small result for the electron mass. But a multibrane scenario has been proposed in [17], where a Randall-Sundrum warp factor enhances the dynamical mass, which then acquires the right order of magnitude for the electron, when M is the Plank mass.

4 Perturbative analysis of the model

Taking into account the fermion mass m_{dyn} , we discuss here the perturbative quantization of the model, first at a one-loop, and we give then the general arguments at higher order loops.

4.1 One-loop properties

Fermion kinetic term

We calculate in the Appendix the one-loop quantum corrections for the fermion kinetic term, which are different for time and space derivatives. We find the corrections

$$i\bar{\psi} \left((1 + Z_0) \partial_0 \gamma^0 - (1 + Z_1) \vec{\partial} \cdot \vec{\gamma} \right) \psi , \quad (20)$$

with

$$\begin{aligned} Z_0 &= -\frac{\alpha}{2\pi} \left(\ln \left(\frac{1}{\mu} \right) + 4 \ln 2 - 2 \right) + \mathcal{O}(\mu^2 \ln(1/\mu)) \\ Z_1 &= -\frac{\alpha}{2\pi} \left(\ln \left(\frac{1}{\mu} \right) + \frac{50}{9} - \frac{20}{3} \ln 2 \right) + \mathcal{O}(\mu^2 \ln(1/\mu)) . \end{aligned} \quad (21)$$

Note that the dominant term, proportional to $\ln(1/\mu)$, is the same for Z_0 and Z_1 , since in the Lorentz symmetric situation ($M \rightarrow \infty$ for fixed fermion mass), we have $Z_0 = Z_1$. Also, the coefficient $-\alpha/(2\pi)$ in front of the dominant term $\ln(1/\mu)$ is the coefficient found in *QED* in $4 - \epsilon$ dimensions

$$Z_0^{QED} = Z_1^{QED} = -\frac{\alpha}{2\pi\epsilon} + \text{finite} . \quad (22)$$

An important remark should be made here: because of the result (19), the ratio μ is actually finite in the limit where $M \rightarrow \infty$, and one could think that no counter term is necessary to absorb terms proportional to $\ln(1/\mu)$. But it is in fact necessary, in order to respect the loop structure. Indeed, if one keeps $\ln(1/\mu)$ -terms in the renormalized theory, the would-be one-loop correction would become a tree-level one:

$$\alpha \ln \left(\frac{1}{\mu} \right) = \mathcal{O}(1) . \quad (23)$$

Therefore, provided one treats $\ln(1/\mu)$ as a $1/\epsilon$ term in dimensional regularization, one gets the usual *QED* one-loop structure.

Maximum speed for fermions

After redefinition of the bare parameters in the minimal subtraction scheme, where only the term proportional to $\ln(1/\mu)$ is absorbed, the fermion dispersion relation is

$$\left(1 - \frac{\alpha}{\pi}[2 \ln 2 - 1]\right)^2 \omega^2 = \left(1 - \frac{\alpha}{\pi}[25/9 - (10/3) \ln 2]\right)^2 p^2 + m_{dyn}^2, \quad (24)$$

and the product of the fermion phase velocity v_ϕ and group velocity v_g is then

$$v^2 \equiv v_\phi v_g = \frac{\omega}{p} \frac{d\omega}{dp} \quad (25)$$

$$\begin{aligned} &= \left(\frac{1 - (\alpha/\pi)[25/9 - (10/3) \ln 2]}{1 - (\alpha/\pi)[2 \ln 2 - 1]} \right)^2 \\ &= 1 - \frac{2\alpha}{\pi} \left(\frac{34}{9} - \frac{16}{3} \ln 2 \right) + \mathcal{O}(\alpha^2) < 1, \end{aligned} \quad (26)$$

which shows that the effective light cone seen by fermions is consistent with causality. If we take $\alpha \simeq 1/137$, we obtain for the fermion maximum speed

$$v \simeq 1 - 1.9 \times 10^{-4}. \quad (27)$$

Gauge invariance and speed of light

We calculate in the Appendix the one-loop vertex for vanishing incoming momentum, and we find the corrections

$$\begin{aligned} \Gamma_{(1)}^0 &= Z_0 e \gamma^0 \\ \Gamma_{(1)}^i &= Z_1 e \gamma^i, \end{aligned} \quad (28)$$

such that gauge invariance is respected, since quantum corrections to the vertex are identical with corrections to the fermion kinetic term, for time and space components independently. As a consequence, as in usual *QED*, quantum corrections to the coupling constant are given by the polarization tensor, and arise only from the wave function renormalization of the gauge field. Since the one-loop polarization tensor doesn't include any internal photon line, the one-loop running of the coupling constant is therefore also the same as in *QED*.

Because $Z_0 \neq Z_1$ and therefore the effective light cone seen by fermions

involves the speed $v < 1$, one might see a problem with the definition of speed of light. We argue here that it is not the case, because of dynamical mass generation for the fermion. Indeed, the speed of light c is defined by

$$c = \lim_{m \rightarrow 0} \frac{\omega}{|\vec{p}|} , \quad (29)$$

for finite momentum \vec{p} and frequency ω . But because the fermion is always massive, $m = m_{dyn} \neq 0$, the limit (29) cannot be taken, and the result (27) is not in contradiction with the speed of light. Such a conclusion was already obtained in [8] for a Lifshitz-type Yukawa interaction.

The speed of light is given by the gauge field dispersion relation, which is not modified at one-loop. We show in the next subsection that this conclusion still holds at higher order loops.

4.2 Higher order loops

Perturbative expansion

We have seen at one-loop that, in order to recover the QED renormalization structure, one needs to absorb terms proportional to $\ln(1/\mu)$ in the redefinition of bare parameters. If we assume that, after including the appropriate counter terms, we have a finite theory at $n - 1$ loops, a graph calculated at n loops will be of the form

$$G^{(n)} = P^{(n)} \frac{M^\epsilon}{\epsilon} + Q^{(n)} \ln \left(\frac{1}{\mu} \right) + \text{finite} , \quad (30)$$

where, compared to $(n - 1)$ -loop graphs:

$P^{(n)}$ contains graphs with an additional fermion loop, which needs to be regularized dimensionally;

$Q^{(n)}$ contains graphs with an additional photon line, which are regularized by M .

The n -loop counter term then needs to absorb terms proportional to both $1/\epsilon$ and $\ln(1/\mu)$. The contribution to the renormalization group flow is then

$$\lim_{\epsilon \rightarrow 0} \left\{ M \frac{\partial G^{(n)}}{\partial M} \right\} = P^{(n)} + Q^{(n)} + \mathcal{O}(\mu \ln \mu) , \quad (31)$$

and corresponds to the usual result obtained in QED , since we have

$$G_{QED}^{(n)} = (P^{(n)} + Q^{(n)}) \frac{M^\epsilon}{\epsilon} + \text{finite} , \quad (32)$$

where the finite terms differ from those in eq.(30). We stress here that the partial derivative with respect to M should be taken at *fixed* fermion mass.

Speed of light

If one considers two-loop properties of the model or higher orders, the polarization tensor is affected by Lorentz violation, unlike in the one-loop case, and it is necessary to make sure that the model remains consistent, especially as far as gauge invariance and speed of light are concerned.

From two loops and above, the field strength gets different corrections for time and space derivatives, and we obtain

$$2(1 + Y_0)F_{0i}F^{0i} + (1 + Y_1)F_{ij}F^{ij} , \quad (33)$$

where Y_0 and Y_1 represent the finite quantum corrections to the operators F_{0i}^2 and F_{ij}^2 respectively, after absorbing the regularization terms proportional to $1/\epsilon$ or $\ln(1/\mu)$. In order to obtain corrections proportional to the Lorentz scalar $F_{\mu\nu}F^{\mu\nu}$, which is necessary to recover gauge invariance and the speed of light $c = 1$, we rescale the time coordinate and the component A_0 as:

$$t \rightarrow \frac{t}{\kappa} \quad \text{and} \quad A_0 \rightarrow \kappa A_0 \quad \text{where} \quad \kappa = \sqrt{\frac{1 + Y_1}{1 + Y_0}} . \quad (34)$$

One can easily see then, that this rescaling is consistent with gauge invariance of the fermion sector:

$$\bar{\psi}(i\partial_0 - eA_0)\gamma^0\psi \rightarrow \kappa \bar{\psi}(i\partial_0 - eA_0)\gamma^0\psi \quad (35)$$

The factor κ , which does not appear in the space components of the covariant derivatives, will then contribute to the maximum speed for fermions, together with the corrections to the fermion kinetic terms, as explained for the one-loop case. A final identical rescaling for all the gauge field components, by the factor $\sqrt{1 + Y_1}$, will lead to the redefinition of the coupling constant.

5 Conclusion

We have shown that Lorentz-violating higher-order derivatives for an Abelian gauge field lead to a fermion dynamical mass, independently of the strength of the coupling constant, and that it also leads to a consistent quantum theory. The order of magnitude of the fermion dynamical mass is too small for

the electron, though, if one identifies M with the Plank mass, and this is a problem which could be solved with the introduction of a non-Abelian gauge field, as explained below.

The main difference with QED is the maximum speed for fermions, which is smaller than the speed of light. This implies that the fermions see a different effective light cone, but it is not in contradiction with the definition of speed of light, since the fermion is always massive.

The next step is to extend this study to non-Abelian gauge theories. It is known that vector fields coupled to the axial current acquire a mass when a fermion condensate forms [18], as the result of the massless bound state excitation, which implies a pole in the polarization tensor for vanishing external momentum. The residue of this pole is the vector mass squared. In the framework of our present model, the fermion condensate can occur, due to Lorentz-violation in the Abelian sector, and then generate dynamically the non-Abelian vector masses. We note that, in order not to break the non-Abelian gauge symmetry at the classical level, a Lorentz-violating non-Abelian dynamics would involve higher order *covariant*-space derivatives, and would therefore introduce new interactions, which are not renormalizable. For this reason, we are planning to leave the non-Abelian sector unchanged, and introduce Lorentz-violation in the Abelian sector only.

Another consequence of the coupling of fermions with several gauge fields will be to change the order of magnitude of the fermion dynamical mass. Indeed, as was already seen in [19], where a similar study was done, the contributions of different gauge fields lead to a partial cancellation in the exponent in the exponential of eq.(19), giving a phenomenologically more realistic dynamical mass.

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Appendix: One-loop Feynman graphs

We calculate here one-loop corrections to the model (1), assuming that the fermion has mass m , which is dynamically generated. The metric we use throughout is $(+, -, -, -)$.

Fermion kinetic terms

The fermion wave function renormalization is obtained by differentiating the fermion self energy with respect to the external momentum ν, \vec{k} , and then set $\nu = 0, \vec{k} = 0$. The fermion self energy is

$$\begin{aligned} \Sigma^{(1)}(\nu, \vec{k}) & \quad (36) \\ &= i(i e)^2 \int \frac{d\omega}{2\pi} \int \frac{d^3\vec{p}}{(2\pi)^3} \frac{\gamma^\mu (\not{p} + \not{k} + m) \gamma_\mu}{(1 + p^2/M^2)(\omega^2 - p^2)[(\omega + \nu)^2 - (\vec{p} + \vec{k})^2 - m^2]} \\ &= -i \frac{e^2}{(2\pi^4)} \int d\omega \int \frac{d^3\vec{p}}{1 + p^2/M^2} \frac{4m - 2(\not{p} + \not{k})}{(\omega^2 - p^2)[(\omega + \nu)^2 - (\vec{p} + \vec{k})^2 - m^2]} . \end{aligned}$$

We write

$$\Sigma^{(1)}(\nu, \vec{k}) = Z_0 \nu \gamma^0 - Z_1 \vec{k} \cdot \vec{\gamma} + \mathcal{O}(k^2) , \quad (37)$$

and a derivative with respect to k^ρ gives, after setting $\nu = 0, \vec{k} = 0$,

$$\begin{aligned} & \left. \frac{\partial \Sigma^{(1)}}{\partial k_\rho} \right|_{k=0} \quad (38) \\ &= \frac{2ie^2}{(2\pi)^4} \int \frac{d^3\vec{p}}{1 + p^2/M^2} \int \frac{d\omega}{\omega^2 - p^2} \left(\frac{\gamma^\rho}{\omega^2 - p^2 - m^2} - \frac{2p^\rho \not{p}}{(\omega^2 - p^2 - m^2)^2} \right) \end{aligned}$$

For $\rho = 0$, we obtain, after a Wick rotation,

$$Z_0 = -\frac{e^2}{2\pi^3} \int \frac{p^2 dp}{1 + p^2/M^2} \int \frac{d\omega}{\omega^2 + p^2} \frac{-\omega^2 + p^2 + m^2}{(\omega^2 + p^2 + m^2)^2} \quad (39)$$

and integration over the frequency ω gives

$$\begin{aligned} Z_0 &= -\frac{e^2}{2\pi^2} \int_0^\infty \frac{x^2 dx}{1 + \mu^2 x^2} \left(2x + \frac{1}{x} - 2\sqrt{1 + x^2} \right) \\ &= -\frac{2\alpha}{\pi} \left(\frac{1}{4} \ln(1/\mu) + \ln 2 - \frac{1}{2} \right) + \mathcal{O}(\mu^2 \ln(1/\mu)), \end{aligned} \quad (40)$$

where $\mu = m/M \ll 1$ and $x = p/m$.

For $\rho = i$, we obtain, after a Wick rotation,

$$Z_1 = -\frac{e^2}{2\pi^3} \int \frac{p^2 dp}{1 + p^2/M^2} \int \frac{d\omega}{\omega^2 + p^2} \frac{\omega^2 + p^2/3 + m^2}{(\omega^2 + p^2 + m^2)^2} , \quad (41)$$

where the constant $1/3$ in factor of p^2 in the numerator comes from the Galilean symmetry in the 3-dimensional space. The integration over ω gives

$$\begin{aligned} Z_1 &= -\frac{e^2}{2\pi^2} \int_0^\infty \frac{x^2 dx}{1 + \mu^2 x^2} \left(-\frac{2x}{3} + \frac{1}{x} + \frac{2x^2/3 - 1}{\sqrt{1 + x^2}} + \frac{x^2/3}{(1 + x^2)^{3/2}} \right) \\ &= -\frac{2\alpha}{\pi} \left(\frac{1}{4} \ln(1/\mu) + \frac{25}{18} - \frac{5}{3} \ln 2 \right) + \mathcal{O}(\mu^2 \ln(1/\mu)) . \end{aligned} \quad (42)$$

Vertex

The one-loop correction to the vertex is

$$\begin{aligned} \Gamma_{(1)}^\rho &= (ie)^3 \int \frac{d\omega}{2\pi} \int \frac{d^3 \vec{p}}{(2\pi)^3} \frac{1}{1 + p^2/M^2} \frac{\gamma^\mu (\not{p} + m) \gamma^\rho (\not{p} + m) \gamma_\mu}{(\omega^2 - p^2)(\omega^2 - p^2 - m^2)^2} \\ &= -\frac{e^3}{4\pi^3} \int \frac{p^2 dp}{1 + p^2/M^2} \int d\omega \frac{2\gamma^\rho(\omega^2 - p^2) - 4p^\rho \not{p} - 2m^2 \gamma^\rho}{(\omega^2 - p^2 - m^2)^2(\omega^2 - p^2)} . \end{aligned} \quad (43)$$

For $\rho = 0$, we obtain after a Wick rotation

$$\begin{aligned} \Gamma_{(1)}^0 &= -\gamma^0 \frac{e^3}{2\pi^3} \int \frac{p^2 dp}{1 + p^2/M^2} \int d\omega \frac{-\omega^2 + p^2 + m^2}{(\omega^2 + p^2)(\omega^2 + p^2 + m^2)^2} \\ &= Z_0 e \gamma^0 . \end{aligned} \quad (44)$$

For $\rho = i$, we obtain after a Wick rotation

$$\begin{aligned} \Gamma_{(1)}^i &= -\gamma^0 \frac{e^3}{2\pi^3} \int \frac{p^2 dp}{1 + p^2/M^2} \int d\omega \frac{\omega^2 + p^2/3 + m^2}{(\omega^2 + p^2)(\omega^2 + p^2 + m^2)^2} \\ &= Z_1 e \gamma^i . \end{aligned} \quad (45)$$

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